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# Polarization, abstention, and the median voter theorem

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The median voter theorem has long been the default model of voter behavior and candidate choice. While contemporary work on the distribution of political opinion has emphasized polarization and an increasing gap between the “left” and the “right” in democracies, the median voter theorem presents a model of anti-polarization: competing candidates move to the center of the ideological distribution to maximize vote share, regardless of the underlying ideological distribution of voters. These anti-polar results, however, largely depend on the “singled-peakedness” of voter preferences, an assumption that is rapidly losing relevance in the age of polarization. This article presents a model of voter choice that examines three potential mechanisms that can undermine this finding: a relative cost of voting that deters voters who are sufficiently indifferent to both candidates, ideologically motivated third-party alternatives that attract extreme voters, and a bimodal distribution of voter ideology. Under reasonable sets of conditions and empirically observed voter opinion distributions, these mechanisms can be sufficient to cause strategically minded candidates to fail to converge to the center, or to even become more polarized than their electorate.

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## Introduction

When is it rational for two strategically motivated candidates to deviate from the ideological center in a general election? Spatial models of economic competition have long served as a baseline model for political agendas and electoral outcomes (Downs, 1957; Hotelling, 1929). In their simplest form, every voter's political preference is captured along a one-dimensional space, and each voter chooses the candidate (typically out of two) who is most proximate to them in the one-dimensional ideological space. Accordingly, each candidate rationally selects a point that maximizes their share of votes. The main result is well-known, two competing and self-interested candidates are at equilibrium when their political positions are equal to the opinion of the median voter.

A simple one-dimensional, two-candidate model of elections ignores many possible complications that have been addressed by political scientists since the conception of the median voter theorem. There may be more than two candidates, or a third option may enter depending on the ideological alignment of the two main candidates (Palfrey, 1984). The ideological space that candidates are competing on may be multi-dimensional (Davis and Hinich, 1966; Hinich and Ordeshook, 1970). Voters may also have probabilistic rather than deterministic voting rules, which can shift the point of candidate ideological convergence from the median to the center (Banks and Duggan, 2005; McKelvey and Patty, 2006). Candidates may not be purely concerned with winning, and gain more utility from winning with a specific ideological position (Kollman et al., 1992).

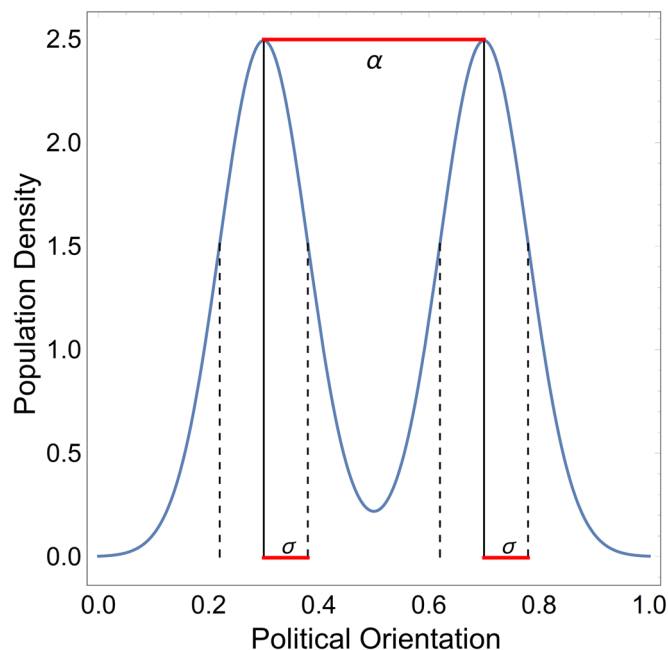
Furthermore, many scholars have found that the polarization of political candidates and other elites leads to the polarization of voters, both in terms of their ideological views (Druckman et al., 2013) and their affective perceptions of the other party (Banda and Cluverius, 2018) and opposing political elites (Rogowski and Langella, 2015). While these mechanisms of top-down social influence are important to consider, voters do not follow polarized elites unconditionally (Mullinix, 2016), and we expect that candidates also have an incentive to follow voters as the original median voter theorem states. In our model we present a set of decision-making rules that could generate voter-driven elite polarization, rather than elite-driven voter polarization or voter-driven political convergence. Stated in another way, we show how polarization could emerge from elite preferences for winning, rather than elite commitments to predetermined ideologies. While voter-driven or elite-driven mechanisms may be sufficient to generate polarization, both are almost certainly at play in “real-world” cases, and that there is substantial interplay between the two. Our modeling approach focuses on the voter-driven model, and presents a parameter space of voter behaviors and distributions. We assess which combinations of behaviors and distributions by themselves would be sufficient to generate polarization between strategically minded candidates.

We focus on a set of three main complications that are undeniably present in the United States, but have not yet been examined together. First, we consider the influence of ideologically motivated third-party candidates. While third-party voting has been on the decline in the United States (Hirano and Jr., 2007), voting for non-competitive third-party candidates still occurs as an expression of cynicism or distrust with the the larger political system (Peterson and Wrighton, 1998), often times at levels that sway the results of major elections (Allen and Brox, 2005). Our models will vary in terms of the intensity of third-party appeal to voters, but the political positions of ideologically motivated candidates will remain fixed at the far ends of the spectrum in our primary model. This is primarily because our model aims to consider centrifugal mechanisms that drive candidates away from the position of the median voter (fixing third-

party candidates also limits the dimensionality of the parameter space in a way that makes interpretation of findings more straightforward). In practice, an ideologically motivated third-party candidate may appear between masses of voters on the left and the right, which we suspect may be an increasingly likely scenario in the United States if polarization increases or persists (we briefly explore the case of an ideologically driven candidate at the political center in section “Candidate optimization against ideological centrism”, and find that under certain conditions this may both prevent the polarization of candidates or prevent their convergence to the center. Our main results and findings, however, will focus on the case of extreme third-parties).

Second, elections in the United States of America typically feature large numbers of eligible voters who stay at home (Franklin, 2004). The reasons for voter abstention are multifaceted, and draw on a range of perspectives from across the social sciences (for a comprehensive review, see (Smets and Van Ham, 2013)). Spatial models of voter choice, where the only voter-level attribute considered is the ideological position of the voter, have considered abstention to be a function of voter-candidate proximity (Anderson and Glomm, 1992; Davis et al., 1970; Enelow and Hinich, 1984; Hinich et al., 1972; Hinich and Ordeshook, 1969; Ordeshook, 1970). Empirical analysis of voting behavior in United States Presidential elections from 1968–80 (Poole and Rosenthal, 1984; Zipp, 1985) and 1980–88 (Adams et al., 2006), as well as United States Senate elections (Plane and Gershtenson, 2004), support the notion that voters are not motivated to take on the cost of voting when they do not find any candidate appealing, or when they are indifferent between candidates. The idea that voters are more likely to show up in response to a higher perceived material or cognitive payoff from a more ideologically proximate candidate aligns with other cost-benefit-based analyses of voting behavior, which find that adverse weather conditions lower voting rates (Gomez et al., 2007), while same-day voter registration decreases costs and increases turnout (Fenster, 1994). We also expect that candidates will disproportionately utilize resources to mobilize voters who are most ideologically proximate and likely to support them. Modern political campaigns that feature repeated targeted attempts at voter contact were found to increase voter turnout by up to 7–8 points in targeted areas during the 2012 United States Presidential Election (Enos and Fowler, 2018). Voter turnout also varies as a function of individual characteristics such as sex, race, and age (Ansolabehere and Hersh, 2011), as well as more mutable socio-demographic factors such as income and wealth (Nadeau et al., 2019), education (Hansen and Tyner, 2021), and health (Blakely et al., 2001; Burden et al., 2017), all of which are conceivably correlated with the relative costs and benefits of voting. Furthermore, voter habit-forming and socialization (Gerber et al., 2003) and social norms (Arceneaux and Nickerson, 2009), will also determine the likelihood of voter turnout or abstention. We expect that net of these social and psychological factors, the ideological positioning of candidates still shape the motives and abstention level of voters. Furthermore, we expect that vote-optimizing candidates will adjust their position to reflect the ideological distribution of typical voters, not the ideological distribution of the entire voting eligible population.

The combined threats of voter abstention and ideologically extreme candidates generate a dilemma between winning over the center or appealing to the “base” when determining what candidates or platforms to field for a general election (Abramson et al., 1992). On one hand extreme candidates might cede the center to the opponent (in line with the assumptions of the median voter theorem), and on the other hand extremist voters may behave irrationally and stay at home rather than casting a



**Fig. 1 Ideological distribution of voters as a function of the two population parameters,  $\alpha$  and  $\sigma$ .**  $\alpha$  is the distance between the two subpopulation centers, and  $\sigma$  is the variance around these subpopulation centers. As  $\sigma$  increases, the population distributions will become less pronounced and more diffuse.

vote for the candidate who is closest to them ideologically. An additional concern is that extremist candidates, while energizing their own base, may increase turnout for people who are extremely opposed to their agenda as well (Hall and Thompson, 2018).

The final mechanism we consider in our model is polarization. Polarization has been examined extensively by political scientists (Fiorina and Abrams, 2008), sociologists (Baldassarri and Bearman, 2007), and economists (Dixit and Weibull, 2007), and its empirical scope and potential causes have been the focus of impressive studies by information scientists (Conover et al., 2011), and computational social scientists (Bail et al., 2018), but its implications for rational choice voting models and candidate competition are rarely considered (Grosser and Palfrey, 2014; Wang et al., 2020). Moreover, median voter-type results often are predicated on the single-peakedness of voter preferences (Black, 1958), which is a problematic assumption during times of polarization. The median voter model can be seen as a “bottom-up” process that brings the political preferences of rational candidates in line with the more centrist preferences of the electorate. It is a model of anti-polarization (Grosser and Palfrey, 2014), but its limitations have become apparent in the current political climate.

Given these three variables: voter tendencies towards third-party candidates, staying home, and polarized beliefs, we are primarily interested in whether specific combinations will motivate strategic candidates to pursue divergent ideological strategies. Given the growing polarization in the U.S. Electorate (Webster and Abramowitz, 2017), it is important to consider the conditions necessary for candidates to follow voters in their drift to extreme positions in the short term. While the use of formal modeling cannot discern the fundamental causality of any specific example of contemporary polarization in the United States or elsewhere, it can shed light on what combinations of voting behaviors could incentivize the drift of candidates away from the center. In practice, we believe this is a more complex phenomenon, likely explained by some combination of intersecting political, cultural,

economic, and technological factors. However, we believe that parsimoniously reconciling this finding with one of the most straightforward, influential, and highly cited models of candidate behavior (Hotelling, 1929; Downs, 1957) spatial choice model) can be generative for future theoretical and empirical research on the intersection of elections and polarization.

Our approach builds on more parsimonious two-candidate models of voter choice, by allowing voters to either choose one of the two main strategically motivated candidates, an ideologically motivated third-party candidate, or stay home altogether. We also consider the ideological distribution of the voter electorate as a proxy for political polarization. Following earlier advances in the voter choice literature, our approach treats voting as a stochastic rather than deterministic process (Coughlin, 1992): the odds of a voter choosing a candidate increase with their relative ideological proximity, but it is never a certainty. This analytical decision is thought to better model voter uncertainty (Burden, 1997). A stochastic voting model has shown that preferential skew does lead to non-median outcomes (Comanor, 1976; Coughlin, 1984; Hinich, 1977), but these models still have one unique equilibrium.

In our analysis, we systematically vary the ideological distribution of voters, the appeal of ideologically motivated third-party candidates on the far ends of the political spectrum, and the appeal of staying home all together. We then map the conditions under which rational political candidates fail to converge on the median ideological position, and also when candidates become more extreme than the electorate itself. We then analyze these dynamics with two empirically observed voter opinion distributions from the contemporary United States.

## Methods and model

**A model of voter selection and population polarization.** Our model examines how a polarized population can influence the political positions of two strategically motivated candidates, who are purely interested in maximizing vote share. Building and integrating the aforementioned models of voter choice, we allow for the possibility that voters may either select an ideologically motivated and extreme candidate instead of a major-party candidate, or that voters may vote for neither candidate if they find their choices unappealing. Third-party candidates in our model are ideologically fixed on either end of the spectrum, as it is presumed that they are motivated by representing a specific position at the far end of the political spectrum, rather than maximizing vote share. Unlike the two rational candidates, they do not shift their position in the model.

Our model considers how both the ideological distribution of the voters and voter tendencies to select one of the two major candidates should influence the political positioning of the two main candidates. These patterns change even when the median and mean voter position is fixed at the center of the distribution. Before discussing the results of our approach, we first outline the two main variable parts of the model: the distribution of the voters and the function that is used to map voter ideology to voter choice and behavior.

**Ideological distribution of voters.** We assume a single-dimensional ideological distribution of voters,  $x$ , on a scale from 0 (left) to 1 (right). We assume that voters made up of two subpopulations, consolidating around two “peaks” that are equidistant from the ideological center (0.5). The distance between the peaks is determined by  $\alpha$ , and the variance in position around these two peaks is determined by  $\sigma^2$ .

Mathematically this provides a population probability density function  $f(x)$  that is the sum of two gaussians centered at  $0.5 \pm \alpha/2$ :

$$f(x) = \frac{c}{\sigma\sqrt{2\pi}} \left[ \exp\left(-\frac{(x - 0.5 - \alpha/2)^2}{2\sigma^2}\right) + \exp\left(-\frac{(x - 0.5 + \alpha/2)^2}{2\sigma^2}\right) \right] \tag{1}$$

Where  $c$  is a normalizing constant to ensure that  $\int_0^1 f(x)dx = 1$ . This population is symmetric, and the median voter is always located at 0.5. Figure 1 illustrates this distribution. While we focus on our model on a hypothetical case where there are two balanced left-leaning and right-leaning subpopulations, the underlying ideological distribution of an actual population, which is not necessarily symmetric, can be calibrated using real voter data from any population of interest (Brown and Enos, 2021).

**Voter choice function.** When voters always select the most ideologically proximate candidate, both parties would still converge to the opinion of the median voter, which is fixed at 0.5. Varying the distribution of voters would have no effect on the strategic ideological positions of the candidates.

Yet in reality political candidates may have concerns about losing their “base” when trying to appeal to the “center”. The threat of losing the base only occurs when voters have the option of either abstaining or selecting a third-party candidate that adapts a position in accordance with their ideology as opposed to vote maximization. Conversely, voters in the center may abstain if both candidates assume positions that are too extreme for them.

In our model, there are three variables that control voting behavior: pragmatism ( $P$ ), which can be thought of as the appeal of voting for a two-party candidate, relative cost of voting ( $Q$ ), which adjusts the voter tendency for staying home, and rebelliousness ( $R$ ), which determines the appeal of third-party candidates.  $P$  and  $R$  are similar, and balance the candidate’s preferences towards an ideologically motivated third-party selection or a more practical two-party selection. When voters are more ideologically equidistant from candidates, they should be more likely to stay home altogether.  $Q$  is a multiplier for this, such that the utility a voter receives from not voting is a product of  $Q$  and a voter’s ideological indifference between the two candidates.

The behavior of the voter is determined by behavioral utilities calculated from the three above parameters, the ideological position of both of the major parties, and the ideological position of the voter in question.

For an individual at  $v$  and major candidates at  $b$  and  $r \in [0, 1]$ , we get the following utilities:

$$\text{Vote Blue Utility} = u_B(b, v) = \frac{1}{|b - v|^P} \tag{2}$$

$$\text{Vote Red Utility} = u_R(r, v) = \frac{1}{|r - v|^P} \tag{3}$$

$$\text{Abstention Utility} = u_A(b, r, v) = (1 - (|b - v| + |r - v|))Q \tag{4}$$

$$\text{Vote Third Party Utility} = u_T(v) = \frac{1}{(1 - v)^R} + \frac{1}{v^R} \tag{5}$$

Each voter chooses from one of the four possible behaviors (vote for red, vote for blue, vote for third party, and abstain) with a probability that is proportional to each of their respective utilities. Figures 2 and 3 provide visual depictions of how voter behavior varies in the model as functions of voter and candidate ideology, respectively.

**Results**

**Voter choice dynamics.** Figure 2 shows voter utilities and corresponding probabilities for a set of parameters. The “candidates” have ideological positions of 0.3 and 0.7, somewhere between being completely polarized and converging to the middle, which roughly reflects two-party elections in the contemporary United States. To illustrate the model, we select a set of parameters for the proposed voter utility functions that lead to an intuitively plausible relationship between voter ideology and voter behavior. The values  $P=2$ ,  $Q=30$ , and  $R=1$  cause more “extreme” voters with an ideology closer to 0 or 1 to be more likely to select a third-party candidate or stay home. Furthermore, the voters in the ideological valley between the two candidates are more likely to stay home, as they do not gain much of a relative benefit from either candidate.

Figure 3 also uses this set of “common sense” decision parameters, but instead focused on the decision behavior of a single voter at a fixed ideological point, and examines how voter behavior corresponds to the ideological positions of the two main candidates. For a voter with an ideology of 0.5, a “median voter”, they become more likely to choose a blue or red candidate when one of them adopts a platform that is ideologically moderate. They become more likely to abstain when both candidates choose more extreme candidate positions on either the same or opposing sides of the political spectrum.

**Candidate optimization.** Given that the behavior of each voter is stochastically determined as a function of their ideological position, the positions of the candidates, and the parameters of our model, we can determine the optimal ideological positions for two competing candidates who are motivated by maximizing vote share. For an ideological space that stretches from 0 (on the left) to 1 (on the right), the liberal and conservative candidates are each seeking an ideological position (“ $b$ ” or “ $r$ ”, respectively) that maximizes the value of one of the following integrals:

$$\text{Blue Votes} = v_B(b, r) = \int_0^1 f(v) \frac{u_B(b, v)}{u_B(b, v) + u_R(r, v) + u_A(b, r, v) + u_T(v)} dv \tag{6}$$

$$\text{Red Votes} = v_R(b, r) = \int_0^1 f(v) \frac{u_R(r, v)}{u_B(b, v) + u_R(r, v) + u_A(b, r, v) + u_T(v)} dv \tag{7}$$

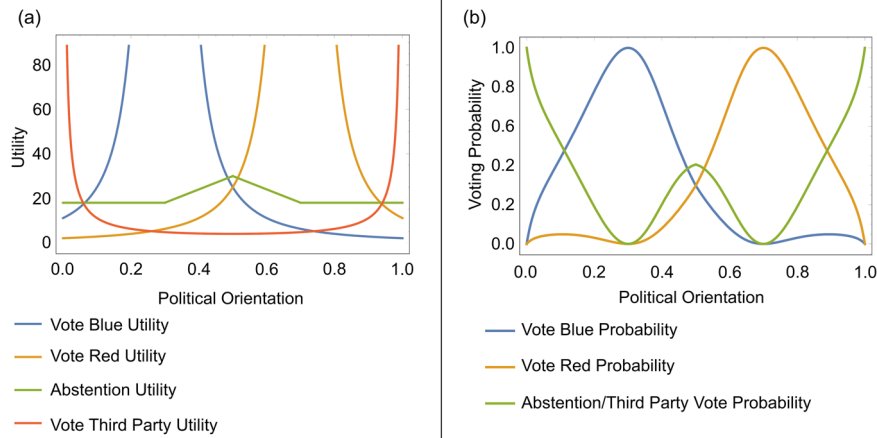
The two major candidates’ theoretical optimization process in response to voters’ behavior can be described by the so-called adaptive dynamics (Hofbauer and Sigmund, 1990; Yang et al., 2020):

$$\begin{aligned} \frac{db}{dt} &= \frac{\partial v_B(b, r)}{\partial b}, \\ \frac{dr}{dt} &= \frac{\partial v_R(b, r)}{\partial r}. \end{aligned} \tag{8}$$

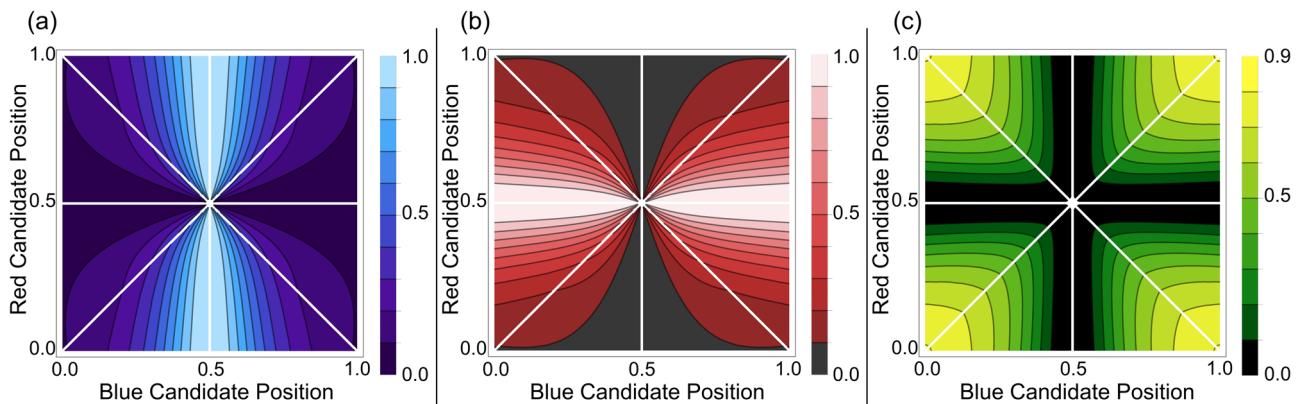
When voters choose the most ideologically proximate of the two competing candidates, both positions converge on the ideology of the median voter. Our model shows how this result does not necessarily hold when voters might choose to abstain or select a third party. In particular this can occur when the distribution of voter preferences is sufficiently bimodal. Figure 4 shows three different sample voter ideological distributions (d, e), and how two political candidates will adjust their ideological platform under a reasonable set of voter choice parameters for each (a–c).

With these three populations, candidate behavior varies from appealing to the median voter when competition is fierce in the high-density middle to being more polarized than the population as candidates work to protect their most extreme voters from a third-party challenge.





**Fig. 2 Voter utility and voting behavior of individuals across the entire political spectrum with two fixed political candidates.** **a** shows the utility that a voter receives from different actions as a function of their position on the political spectrum, assuming candidate positions of 0.3 and 0.7 and a specific set of model parameters ( $P=2, Q=30, R=1$ ). **b** Maps these utilities into one of three behaviors: voting for the “blue” (left-leaning) candidate, the “red” (right-leaning) candidate, or voting for neither (staying home or selecting an ideologically motivated third-party candidate).

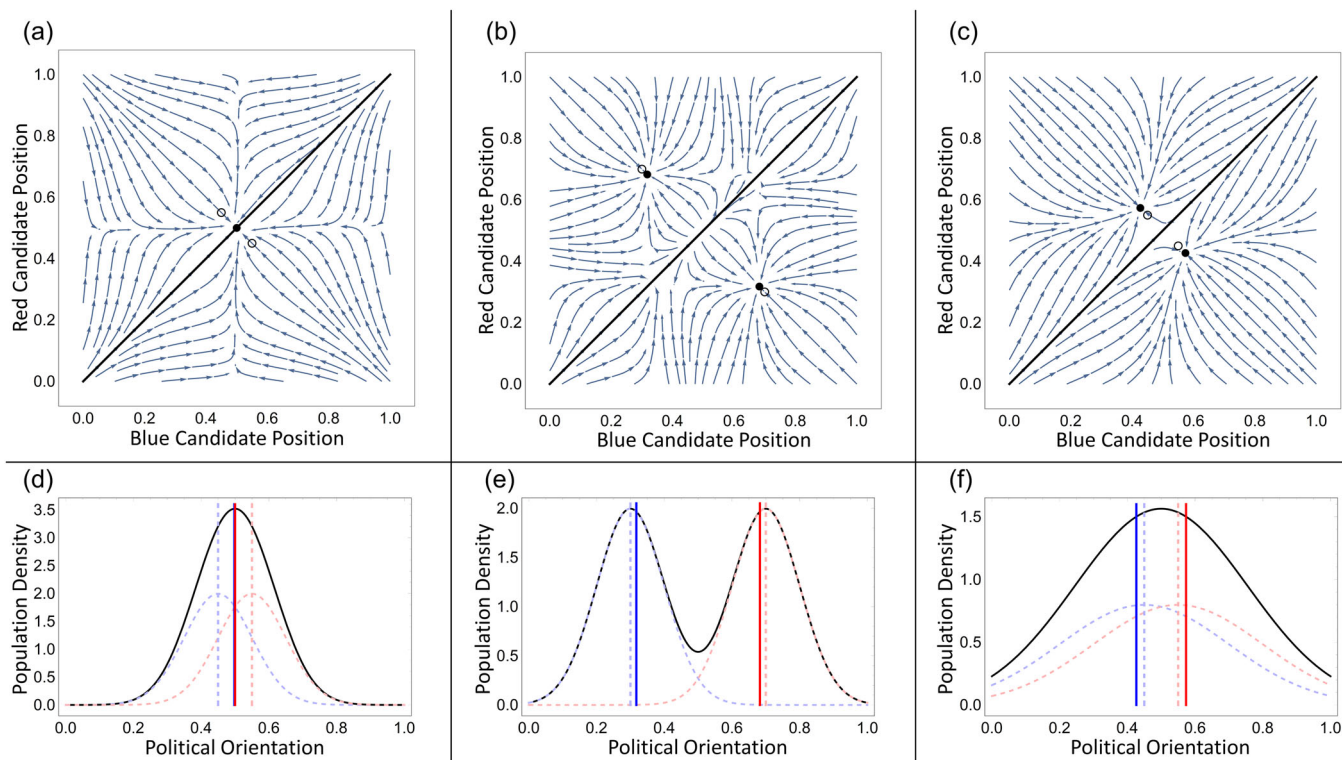


**Fig. 3 Varying the candidates positions influences a fixed voter's behavior.** Each panel shows how the likelihood of a given voter behavior (voting “blue” in **a**, voting “red” in **b**, or voting for neither in **c**) changes as a function of the two political candidates stated ideological position,  $[0, 1]^2$ , assuming a voter ideology of 0.5, a specific set of model parameters ( $P=2, Q=30, R=1$ ). (The white lines crossing the space and point at the center reflect one-dimensional portions of the two-dimensional space where a candidate has the exact same preference as a voter ( $r=0.5$  or  $b=0.5$ ), where the two candidates are identical ( $r=b$ ), or where the voter is equidistant between the two candidates ( $|r-0.5|=|b-0.5|$ ). In these subsets of the space, and their intersection at  $b=0.5=0.5$  the model loses practical interpretability for this single hypothetical voter.

*Candidate optimization against ideological centrism.* While our base model works under the assumption that ideologically motivated candidates come from the far ends of the political spectrum, it is also possible that an ideologically fixed third-parties could run from the center. We examine the effects this has on candidate optimization under the parameter setting  $P=5, Q=0$ , and  $R=5$  in Fig. 5. The results are both intuitive and counterintuitive. Unsurprisingly, major-party candidates in a bimodal population are pulled closer to the center than under conditions of extreme third-party candidates. However, in a unimodal population, major-party candidates are pushed towards the fringes by the centrist third-party, as they benefit from distinguishing themselves from the centrist. This suggests that under a certain set of voter behaviors, having an ideologically fixed third-party candidate may be a solution to both the problems of runaway extremism of polarization and the stasis of convergence to the center. While the remainder of our analysis continues to focus on the case of ideologically extreme third-parties, this insight may present a promising avenue for future empirical and theoretical research.

**Candidate positions and voter distributions.** Depending on voter predisposition to extremist third-party candidates, or their willingness to simply stay home in the absence of an appealing candidate, the rational positions taken by main candidates will vary. In our model, candidates qualitatively do one of three things. They either (1) converge to the median, (2) deviate from the median but still select positions between the two peaks of public opinion, or (3) deviate from the median to a greater extent than the voting base. Two examples of how voter ideology distribution shapes candidate positions is shown in Fig. 6. For each selected set of sample model parameters, each of the three possible candidate outcomes are possible depending on the ideological spread of voters.

While possibility (2) is interesting primarily because of its deviation from the results typically derived by the median voter theorem, possibility (3) reveals a potential long-term mechanism for voter polarization. While our model assumes that voter preferences are static and the position of strategic candidates are dynamic, other models have considered the possibility that voter positions eventually come to resemble candidate positions



**Fig. 4 Optimal behavior of two competing major candidates in different population distributions.** **a-c** show how candidates will shift their position in a stream plot, with the black dot representing the candidates' equilibria, and the circles showing the subpopulation peaks. **d-f** show the corresponding populations in black, the two subpopulations with dashed curves, the subpopulation centers represented by dashed vertical lines, and the candidate equilibrium positions represented by solid vertical lines. We see three types of behavior: candidates converging to the median voter (**a, d**), candidates less polarized than the population (**b, e**), and candidates more polarized than the population (**c, f**). All plots use a reasonable set of parameters  $P = 2$ ,  $Q = 30$ , and  $R = 1$ .

(Kollman et al., 1992). If voter behavior and ideological distribution is one that motivates extremism among rational candidates, this may in turn create a larger spread among voters.

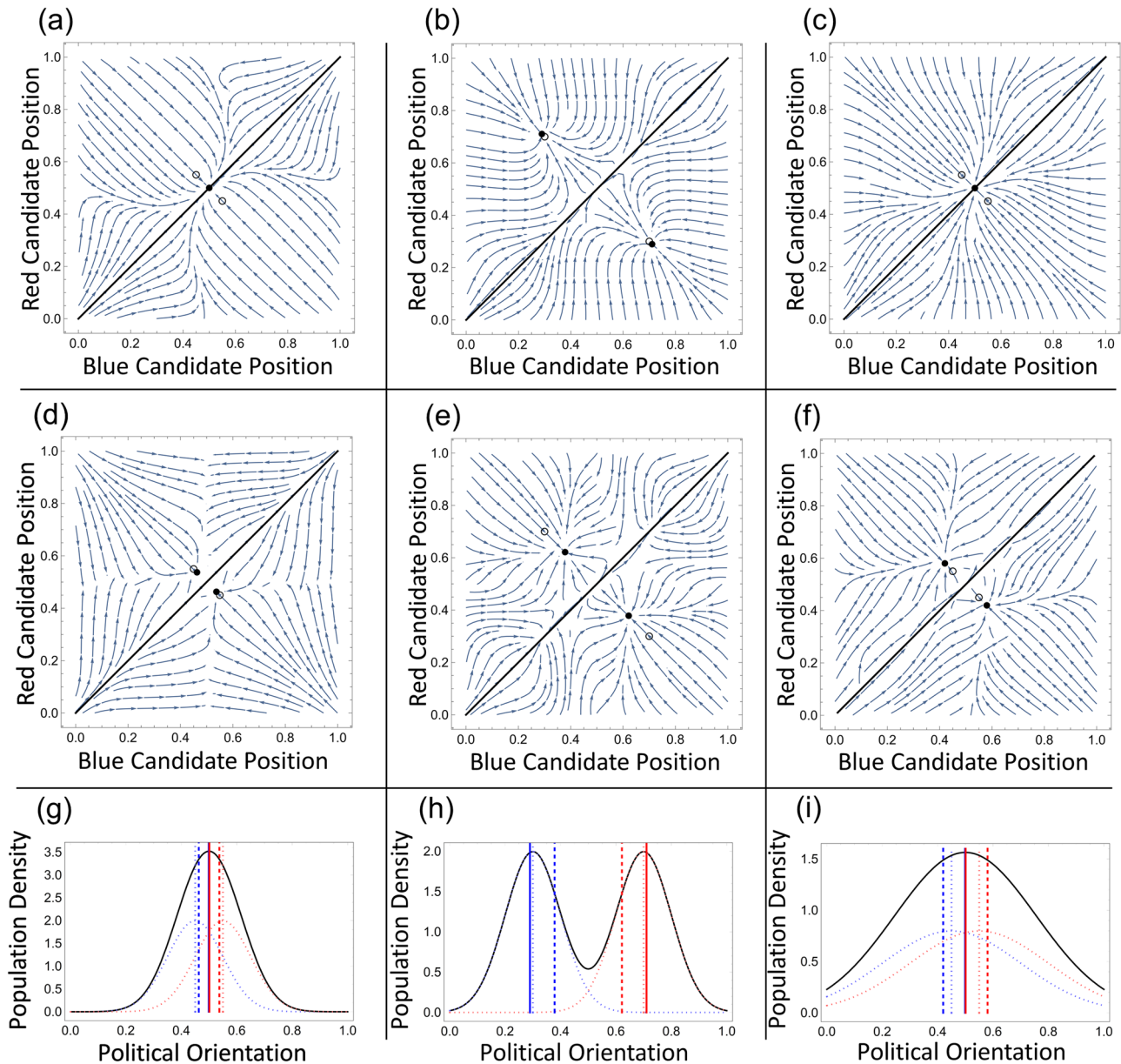
The model parameters have a strong influence on how the ideological structure shapes the strategic polarization of the two main candidates. In Fig. 7, we see how changing one of the three parameters,  $P$ , changes candidate position relative to ideological distribution of the population. With lower levels of  $P$ , voters are less likely to “settle” for a major candidate, and accordingly the candidates must drift from one another to capture more extremist voters, although this split does not exceed the bimodal “peaks” in the underlying population, unless variance around the peaks is high relative to variance between the peaks. Higher levels of  $P$  mean that voters are more pragmatic and tend to favor voting for the two major parties. They are less likely to see the parties diverge from one another, but when distance between the ideological peaks is high, the parties may be on the extreme sides of the two population centers, and candidates then need to capture vote share from extremist third-parties.

*Visualizations of ideological structures and candidate positions for varying values of  $P$ ,  $Q$ , and  $R$ .* A challenge in making sense of our theoretical model and results is that there are five dimensions that can be varied: three correspond to the decision-making of individual voters ( $P$ ,  $Q$ , and  $R$ ), and two correspond to the ideological distribution of voters ( $\sigma$  and  $\alpha$ ). In Fig. 8 we examine this space more comprehensively. We explore the full range of values for  $\sigma$  and  $\alpha$ , and both “high” and “low” values of  $P$ ,  $Q$ , and  $R$ . This is shown in a series of eight two-dimensional plots, colored by

whether the two strategic parties converge to the center, split but remain more centrist than the population centers, or split in a way that exceeds the ideological divergence of the population centers.

This visualization lends itself to broader insights, some obvious and some less so. The two parties are more likely to converge when the ideological centers of the population structures are closer together (lower  $\alpha$ ), although this is not the case when the cost of voting or the appeal of third-parties is high. In general, when the appeal of third-party candidates is low and the cost of voting is low (meaning most voters show up to the polls), assumptions that align more closely with earlier spatial voting models, convergence to the center is ubiquitous. Under more unimodal ideological distributions (with high ratios of spread around ideological centers compared to the split between the centers) the candidates converge to the center, and with more bimodal ideological distributions (low ratios of spread to split) the candidates still become more moderate than the population centers. The latter type of result deviates from the traditional median voter result because voters do not choose their most proximate candidate with certainty. Similar results hold when the cost of voting is high, but mainstream parties have stronger appeal.

Another outcome of interest is when the two parties diverge to a greater extent than the underlying population. This occurs under two combinations of conditions: (a) when the ideological distance between population centers ( $\alpha$ ) is low, the spread of voters around the population centers ( $\sigma$ ) is high, and the cost of voting ( $Q$ ) and/or third-party appeal ( $R$ ) is high, or (b) when the



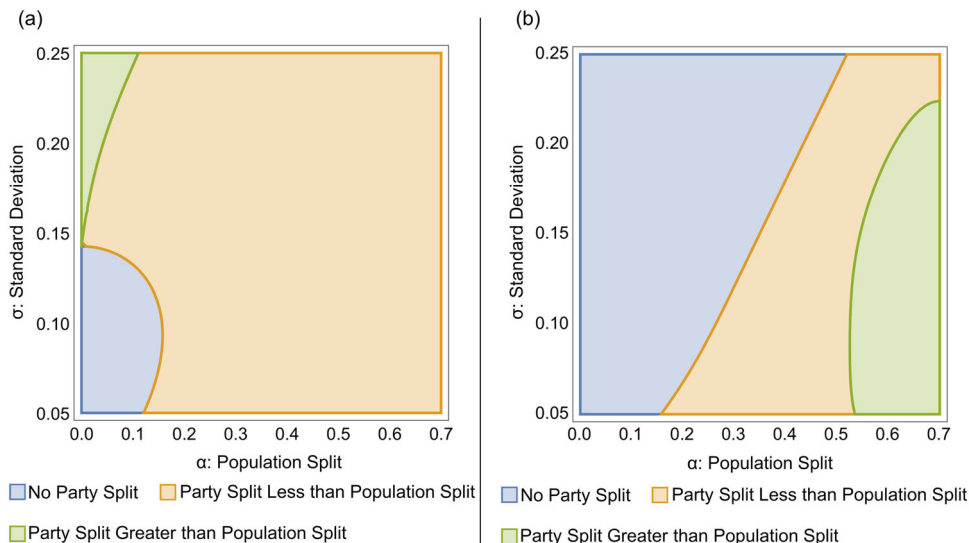
**Fig. 5 Optimal position of two competing major candidates with different third-party behavior.** **a-c** show candidate behavior with two extreme third-parties, and **(d-f)** correspond to a single third-party located at 0.5. The black dots represent the candidates' equilibria and the circles show the subpopulation peaks. **g-i** show the corresponding populations in black, the two subpopulations with dashed curves, the subpopulation centers represented by lightly colored and dashed vertical lines, the candidate equilibrium positions in the extreme condition represented by darkly colored and solid vertical lines, and candidate equilibrium positions in the centrist condition represented by darkly colored and dashed vertical lines. When the population is bimodal **(h)**, centrist parties “pull-in” main candidates from the ideological peaks **(b)** instead of matching the ideological peaks of the voters **(e)**. However, when the population is more unimodal **(g, i)**, centrist third-party candidates either prevent main-parties from completely converging to the center **(d)** or push them away from the center **(f)**, compared to extreme third-party candidates **(a, c)** who cause the main-parties to converge to the center. All plots use the set of parameters  $P = 5$ ,  $Q = 0$ , and  $R = 5$ .

distance between the population centers ( $\alpha$ ) is high, the spread around the population centers ( $\sigma$ ) is low, the third-party appeal ( $R$ ) is high, and the appeal of the main-parties ( $P$ ) is low. Condition (a) corresponds to a situation where there is a nearly unimodal but somewhat ideological diverse electorate that is reluctant to vote or drawn to third-parties, main-parties must contend with apathy and third-party appeal by moving away from the center and distinguishing themselves from one another. Condition (2) corresponds to a situation where a highly and uniformly polarized electorate is drawn towards third-parties so main-parties will win few converts from the other side, and

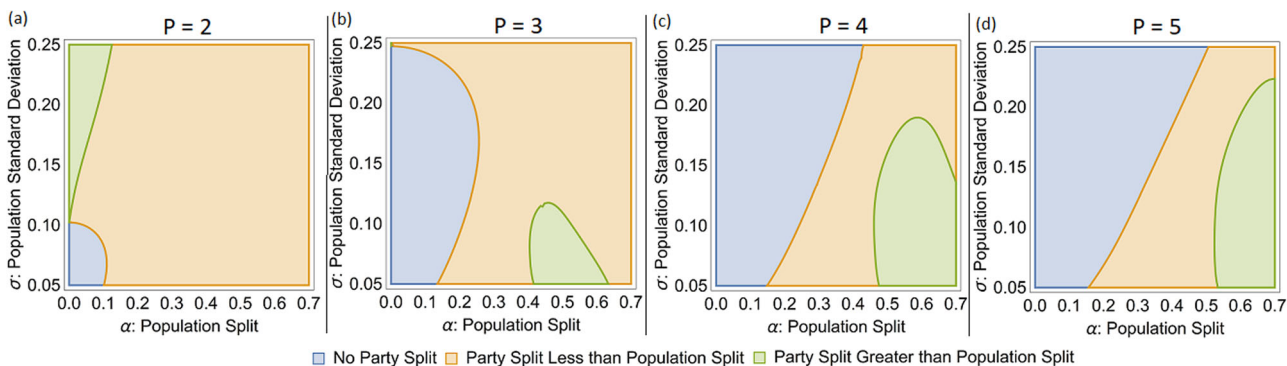
instead try to mobilize the extreme wings of their corresponding side of the ideological voter distribution.

*Incorporating empirically observed ideological distributions.* The results of our model show that under the very basic assumptions of voters being attracted to third-party candidates or being prone to staying home, it may make sense for candidates to avoid the center depending on the distribution of voter ideology. We can incorporate observed empirical distributions of voter opinions into a set of model parameters ( $P = 2$ ,  $Q = 30$ , and  $R = 1$ ) to examine how this model of voter choice might function under





**Fig. 6** For two sets of model parameters, the nature of the equilibrium candidate positions is shown as a function of the distribution of voter ideology. In both plots, the x-axis is population split and the y-axis is the standard deviation of the two subpopulations. **a** uses parameters  $P = 2$ ,  $Q = 30$ ,  $R = 1$ , and **b** uses  $P = 5$ ,  $Q = 0$ , and  $R = 5$ . The three regimes of interest are whether candidates converge to the same position (roughly the mean/median voter theorem result), whether they separate to a lesser extent than the population (the space between the two peaks), or whether they separate to a greater extent than the population. Each space is shaded by adherence to one of these three regimes.



**Fig. 7** Sets of model parameters where  $Q = 30$  and  $R = 5$ , and  $P$  takes on values from 2 to 5. As  $P$  increases, the voter choice model gives more weight to “pragmatic” main party opinions, and the mapping between ideological distribution and the relative political position of the two main-parties changes. When  $P$  is lower, political parties diverge under most ideological distributions, but only more than the public itself when there is high variance around two otherwise close ideological centers. When  $P$  is higher, parties converge to the middle when the population split is low, and are more polarized than the electorate when the population split is large, but the variance around the two ideological centers is low.

contemporary ideological distributions in the United States. Our empirical voter distributions come from two sources. In Fig. 9a, we see the first data set from (Pew Research Center, 2017). As we can see, the population here is neither symmetric nor bimodal. However, there is still enough spread in the distribution of the voters to generate a separation between two candidates. The true median of the population ideology is roughly 0.42, but candidates converge to positions at about 0.25 and 0.51. Perhaps unsurprisingly, the asymmetric distribution of voter preferences leads to differing distances between the median position and each of the candidates.

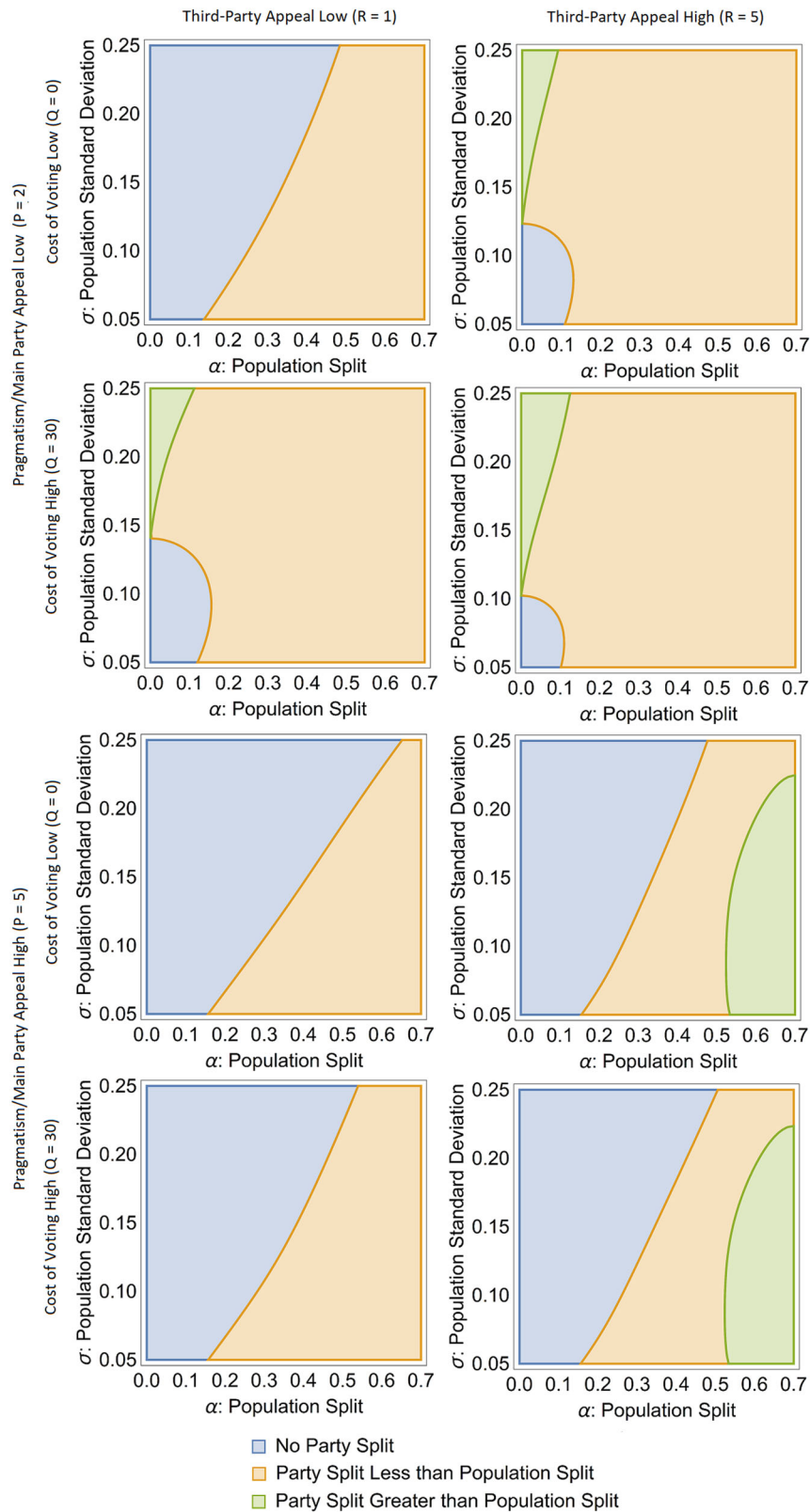
Data from the estimated ideological positions of Twitter users provides a more polarized empirical distribution for examination, and was taken from Figure 3 of (Mukerjee et al., 2020) by means of redigitalization. In Fig. 9c we can see that there is a more roughly bimodal distribution, although it remains asymmetric. While we unfortunately cannot derive values of model parameters  $P$ ,  $Q$ , and  $R$  without data on how individual opinions map into votes, we can look at how candidates should behave strategically

under the observed empirical distributions and the previously used parameters of  $P = 2$ ,  $Q = 30$ , and  $R = 1$ . Once again, political candidates converge on positions that deviate from the median of roughly 0.57, with the left-leaning candidate selecting a position all the way at 0.2, and the right-leaning candidate selecting a position at 0.65. Curiously, while there are more voters on the right than on the left here, the far left positions of the left half of the distribution bring the left-leaning candidate very far from the median. If we suppose that in equilibrium the left choice would win half of the time and the right choice would win half the time, the average position of the winning candidate would be roughly 0.38, very far to the left of the median of 0.57. In this case, the willingness of voters to abstain or vote “irrationally” for third-party candidates gives more weight to the side that entertains more extreme positions.

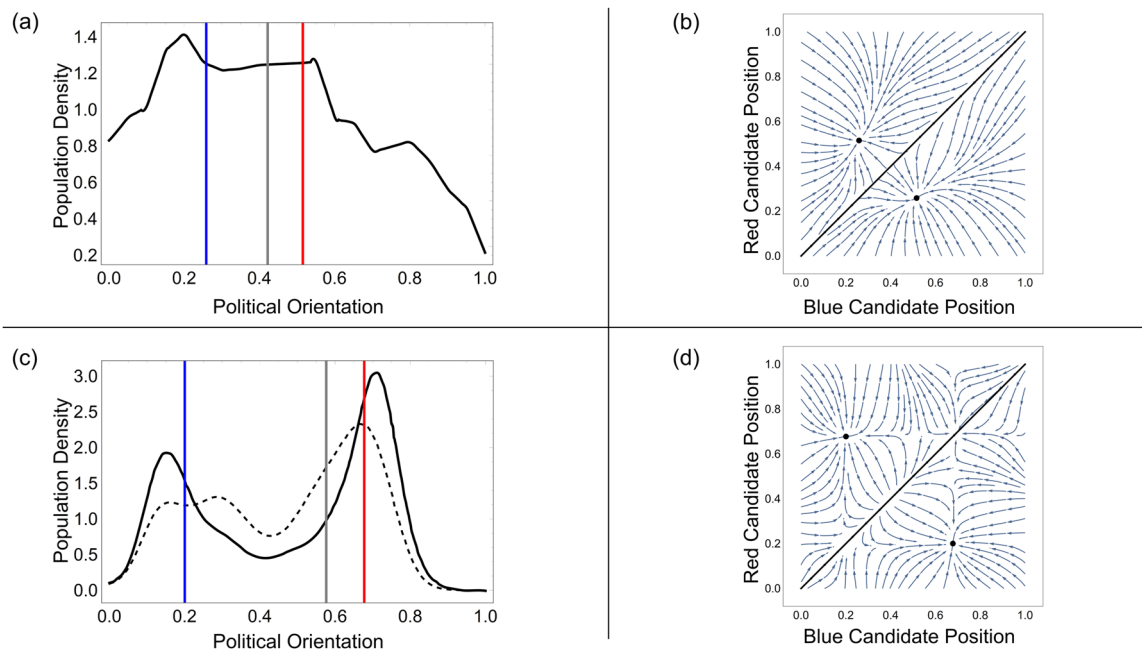
**Discussion**

The assumption that voters are purely rational is a strong one. There is strong evidence that collective opinion dynamics are





**Fig. 8** The positioning of political parties relative to ideological population centers for different parameters of the voter choice model ( $P, Q, R$ ) and different population ideological distributions ( $\sigma, \alpha$ ).  $P$  takes values of 2 (rows 1 and 2) and 5 (rows 3 and 4),  $Q$  takes values of 0 (rows 1 and 3) and 30 (rows 2 and 4), and  $R$  takes values of 1 (left column) and 5 (right column).  $P$  and  $R$  seem to have the most impact on optimal candidate behavior, with  $Q$  only changing behavior when  $P$  and  $R$  are both small.



**Fig. 9 Candidate optimizations based on real-world ideological distributions.** Here we show the results of testing our model on real-world data. First, we used the distribution of voter ideology according to the Summer 2017 Political Landscape Survey (Pew Research Center, 2017) (a), and the rational candidate responses to this landscape (b). We also used an analysis of Twitter users (Mukerjee et al., 2020) (c), and the rational candidate responses to this landscape (d). In a and c, vertical lines show the convergent position of each candidate (blue and red) and the position of the median voter (gray). c also shows the political ideology of political leaders on twitter in dashed gray, and we see that the vertical blue and red lines match nicely with peaks for this curve. Both models used  $P = 2$ ,  $Q = 30$ , and  $R = 1$ , a set of parameters that approximate the credence that an average voter may give to not voting or voting for a third party. The population in c can be fit to a bimodal distribution. The best fit has the left and right subpopulations' peaks at 0.18 and 0.70, standard deviations at 0.09 and 0.07, and relative weights at 1 and 1.32, respectively. In a, the population is less bimodal, and so the fit has less value. However, for completeness, we give the values here: positions 0.18 and 0.37, standard deviations 0.04 and 0.41, and relative weights of 1 and 45.

shaped by processes of social influence (Castellano et al., 2009; Flache et al., 2017; Noorazar, 2020), and voting behavior is no exception (Redner, 2019). Here, we assume that voter irrationality is captured by their opinion formation process, and we instead consider how candidates should rationally respond to different levels of voter polarization and indifference. There is no shortage of proposed mechanisms that explain why voter attitudes have become more polarized over the years. Attitude polarization can result from the twin-mechanisms of homophily, a phenomenon that spans the social and biological sciences (Fu et al., 2012; McPherson et al., 2001), and social influence or the diffusion of pairs of associated beliefs (DellaPosta, 2020; DellaPosta et al., 2015; Goldberg and Stein, 2018). The programming decisions of large media outlets (Campante and Hojman, 2013; Prior, 2013; Sunstein, 2018), and the recommendation algorithms of social media sites can send people into wildly different information landscapes (Barberá, 2020; Levy, 2021). The influence these social processes may have on political candidates, however, is less examined. Our model shows that there are very realistic conditions under which rationally behaving major-party candidates will benefit from reinforcing polarization rather than by pivoting to the center.

This approach, like any model, is limited by the complexity that it omits. Regarding the specifics of voting in the United States, it omits details on the primary process and how candidates may be bound by verbal commitments they made to a primary electorate while running a general campaign. It omits the possibility of a serious third-party entering the race with strategic rather than ideological motives. It omits the draw that candidate personality may have on the behavior of voters. Moreover, it assumes a linear single-dimensional model of ideological

positions rather than a multi-dimensional space (Yang et al., 2020). It also omits the institutional and geographic complexities of voting induced by district or state-based electoral systems combined with the tendency for voters to self-sort geographically (Martin and Webster, 2020), and strategic attempts to manipulate this process such as gerrymandering (Stewart et al., 2019). Furthermore, our model assumes that voters have independent, static, and rational voting preferences. The study of collective opinion dynamics often focuses on the role of social influence, and voting behavior is no exception. Our model assumes a pre-polarized and static distribution of voter preferences, a simplifying assumption that allows us to focus on the rational behavior of candidates who are trying to win over a polarized electorate.

Yet, the minimal number of realistic assumptions necessary to obtain this result makes it all the more compelling and concerning. Stochastically determined voters with a bimodal ideological distribution and the option to not vote for a major candidate may incentivize more extreme political parties. If we are to believe that voters follow candidates and parties just as candidates and parties follow voters, then a distributional tipping point may exist where voters and candidates chase each other to ideological extremes (see SI for an explicit analysis of such tipping point of population split  $\alpha$ ). The solutions to this problem may be found in practices not explored in this model. For example, as we briefly explored in section "Candidate optimization against ideological centrism", ideologically motivated candidates running from the center may effectively "pull in" extreme but strategic candidates, in the same way that ideologically motivated extremist candidates can pull strategic candidates away from the center. The polarized political climate in the United States (and elsewhere) remains a serious problem, and continued reconsideration of rational choice

voting models with more contemporary assumptions may provide the theoretical material necessary to develop pragmatic solutions for ending what is being referred to by some as a “cold civil war” (Kay, 2021).

### Data availability

All the data and analysis pertaining to this work have been included in the main text. This article has no additional data.

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### Competing interests

The authors declare no competing interests.

### Ethical approval

Ethical assessment is not required prior to conducting the research reported in this paper, as the present study does not have experiments on human subjects and animals, and does not contain any sensitive and private information.

### Informed consent

Informed consent is not required prior to conducting the research reported in this paper, as the present study does not have experiments on human subjects, and does not contain any sensitive and private information.

### Additional information

**Supplementary information** The online version contains supplementary material available at <https://doi.org/10.1057/s41599-022-01056-0>.

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